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Bias correction for non-stationary noise filtering in MRI

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In a nutshell

Problem:

- The aggregation of non-stationary distributed magnetic resonance imaging (MRI) samples results in a systematic bias.

Solution:

- We analytically derive closed-form formulas to compensate the bias from aggregated non-stationary non-central chi (nc-χ) distributed random variables (RVs).
- We reformulate the unbiased non-local means (UNLM) scheme to handle non-stationary nc-χ (Rician) distributed MRI data.

Non-stationary nc-χ noise

The composite magnitude signal $M(j)$ in parallel accelerated MRI can be modelled using a non-stationary nc-χ distribution with the noise variance $\sigma^2(j)$ and the number of receiver coils $L(j)$.

Method	Noise parameters
SENSE	$\sigma(j)$, $L(j) = 1$
GRAPPA+SMF	$\sigma(j)$, $L(j) = 1$
GRAPPA+SoS	$L_{\mathrm{eff}}(j)$, $\sigma_{\mathrm{eff}}(j)$

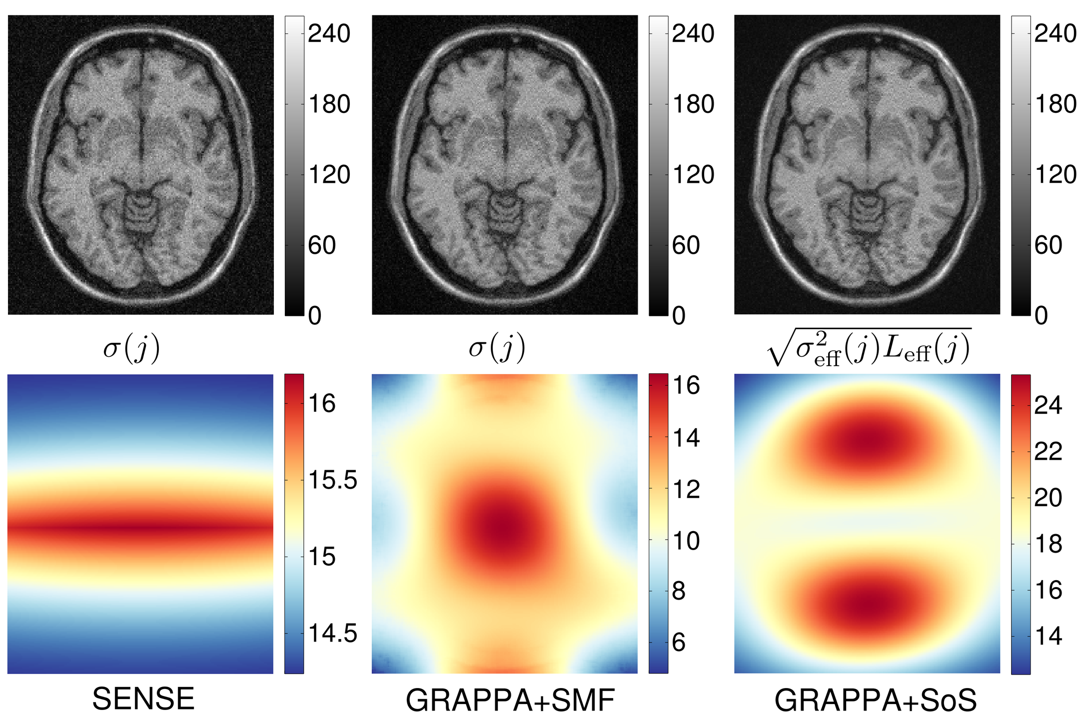

Figure 1: Noise patterns for composite magnitude signal $M(j)$ in typical parallel accelerated MRI acquisitions.

Unbiased non-local means (UNLM)

The filtered image using UNLM can be obtained as:

$$\mathrm{UNLM}(i) = \left(\max \left\{ \sum_{j \in \mathcal{V}(i)} w(i,j) M^2(j) - 2L\sigma^2(i), 0 \right\} \right)^{1/2}$$

- $\mathcal{V}(i)$ – the search window,
- $w(i,j)$ – the weight assigned to $M(j)$.

The distance between the patches $\mathcal{N}(i)$ and $\mathcal{N}(j)$

$$\mathrm{dist}\big(\mathcal{N}(i), \mathcal{N}(j)\big) = \left\| \sqrt{\mathcal{G}_{\Sigma}} \big(M(\mathcal{N}(i)) - M(\mathcal{N}(j)) \big) \right\|_2^2$$

with $\mathcal{G}_{\Sigma}$ being a circularly symmetric Gaussian kernel.

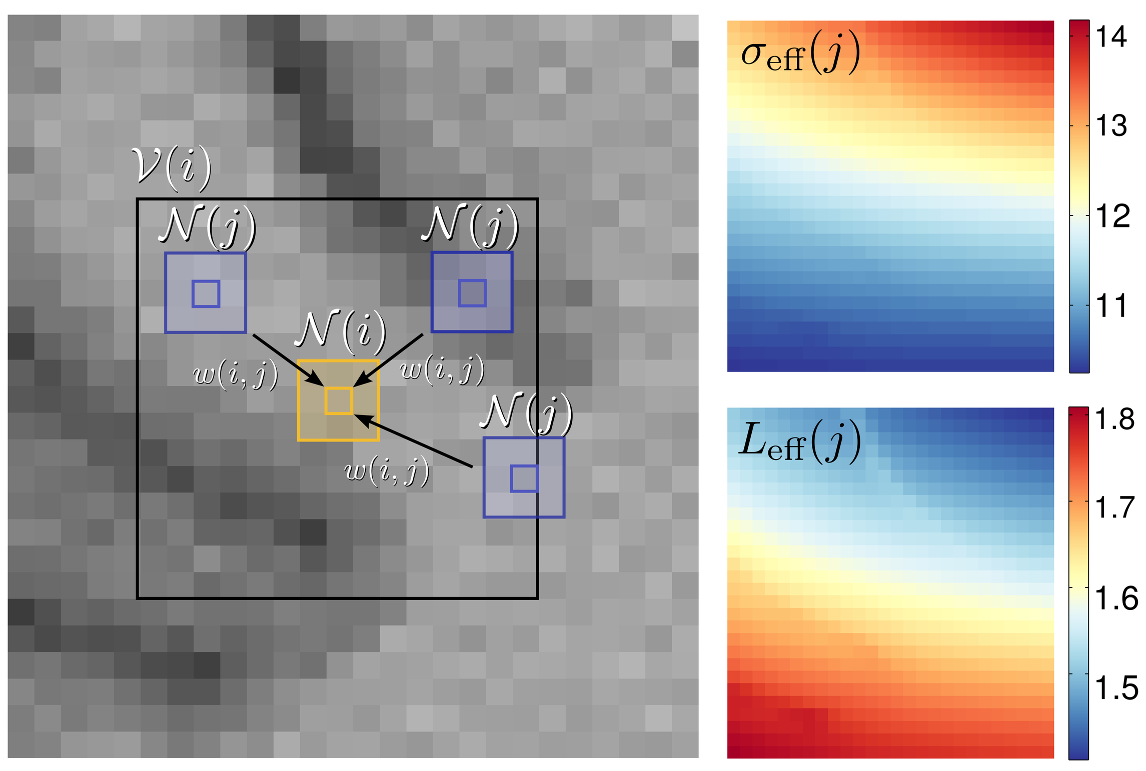

Figure 2: The NLM scheme and the parameters $\sigma_{\mathrm{eff}}(j)$ and $L_{\mathrm{eff}}(j)$.

Second-order bias correction

Let us assume that the filtering of MR data is done using the sum of the weighted squared signals $M(j)$ with corresponding weights $w(j)$

$$\mathrm{S}_2 = \sum_{j=1}^{N} w(j) M^2(j), \quad w(j) \geq 0, \qquad (1)$$

where $M(j)$ follows a nc-χ distribution.

If we assume the samples to be independent, the expectation of (1) becomes:

$$\mathbb{E}\{\mathrm{S}_2\} = \sum_{j=1}^{N} w(j) A_L^2(j) + 2 \sum_{j=1}^{N} w(j) L(j) \sigma^2(j). \qquad (2)$$

This aggregation of non-identically distributed RVs $M^2(j)$ leads to a positive bias depending on the weights $w(j)$ and the parameters $\sigma(j)$ and $L(j)$.

Assuming $\widetilde{A}_L^2 = \sum_{j=1}^{N} w(j) A_L^2(j)$ and replacing the expectation operator by its weighted sample in eq. (2), we reformulate the UNLM as follows:

New solution

$$\text{ns-UNLM}_2(i) = \left(\max \left\{ \sum_{j \in \mathcal{V}(i)} w(i,j) M^2(j) - 2 \underbrace{\sum_{j \in \mathcal{V}(i)} w(i,j) L(j) \sigma^2(j)}_{\text{correction factor}}, \ 0 \right\} \right)^{1/2}.$$

For $L(j) = 1$ and $\sigma(j) = \sigma$, the ns-UNLM$_2$ reduces to the state-of-the-art UNLM for stationary Rician noise.

First-order bias correction

Alternatively, we now assume that the filtering is done using a weighted averaging of the samples $M(j)$

$$\mathrm{S}_1 = \sum_{j=1}^{N} w(j) M(j), \quad w(j) \geq 0. \qquad (3)$$

Using the asymptotic expansion of the first raw moment of nc-χ distributed RV $M(j)$, the expectation of S_1 turns into:

$$\mathbb{E}\{\mathrm{S}_1\} = \sum_{j=1}^{N} w(j) A_L(j) + \frac{1}{2} \sum_{j=1}^{N} w(j) \frac{(2L(j)-1)\sigma^2(j)}{A_L(j)}. \qquad (4)$$

This bias incorporates underlying signals $A_L(j)$.

Assuming $\widetilde{A}_L = \sum_{j=1}^{N} w(j) A_L(j)$, replacing the expectation operator in eq. (4) by a sample estimator and simplifying our considerations to $A_L(j) = \widetilde{A}_L$ we get the closed-form solution:

New solution

$$\text{ns-UNLM}_1(i) = \frac{1}{2} \left[\sum_{j \in \mathcal{V}(i)} w(i,j) M(j) + \left(\max \left\{ \left(\sum_{j \in \mathcal{V}(i)} w(i,j) M(j) \right)^2 - 2 \underbrace{\sum_{j \in \mathcal{V}(i)} w(i,j) \left(2L(j) - 1\right) \sigma^2(j)}_{\text{correction factor}}, \ 0 \right\} \right)^{1/2} \right].$$

For high SNRs, the expression for ns-UNLM$_1$ reduces to the state-of-the-art NLM approach.

Experimental results

T_1-weighted MR data

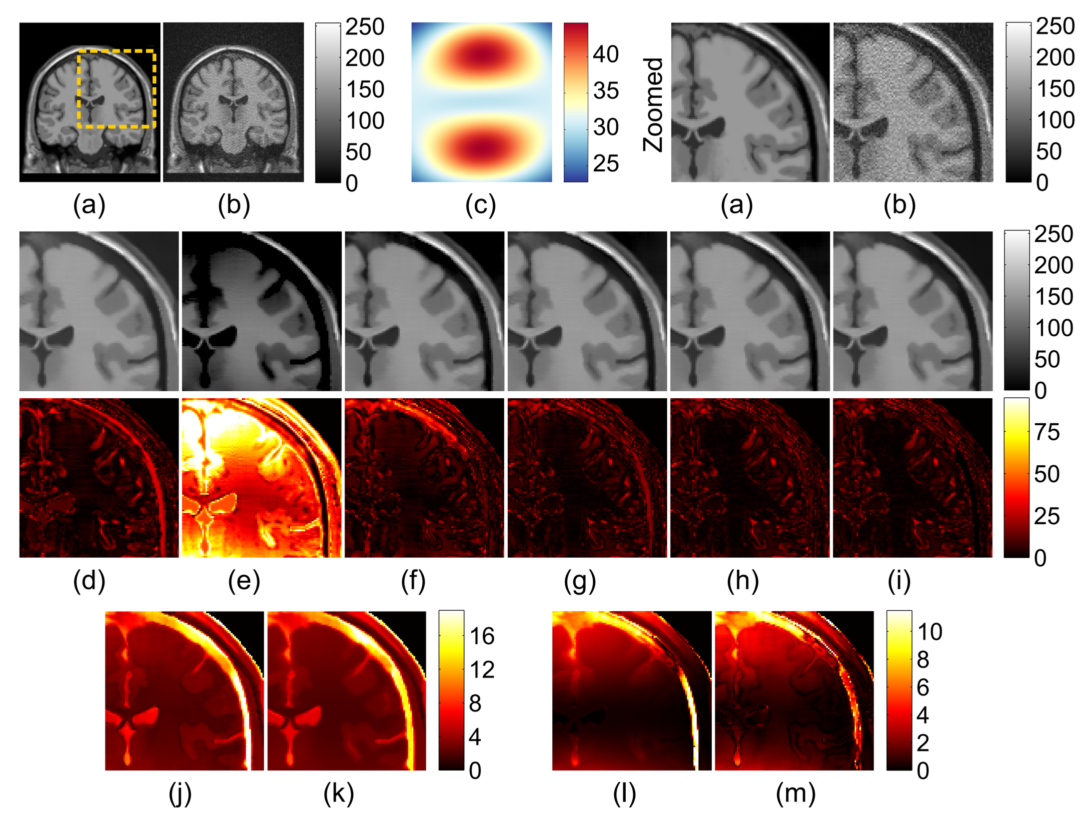

Figure 3: (a) Reference, (b) noisy and (c) noise pattern. Denoised: (d) NLM, (e) UNLM $L(j) = 8$, (f) UNLM $L(j) = 2$, (g) UNLM $L(j) = 1$, (h) ns-UNLM$_2$, (i) ns-UNLM$_1$.

Third row: absolute errors of the methods. Fourth row: absolute differences between (j, k) NLM and ns-UNLM$_2$, ns-UNLM$_1$ and (l, m) UNLM $L(j) = 1$ and ns-UNLM$_2$, ns-UNLM$_1$.

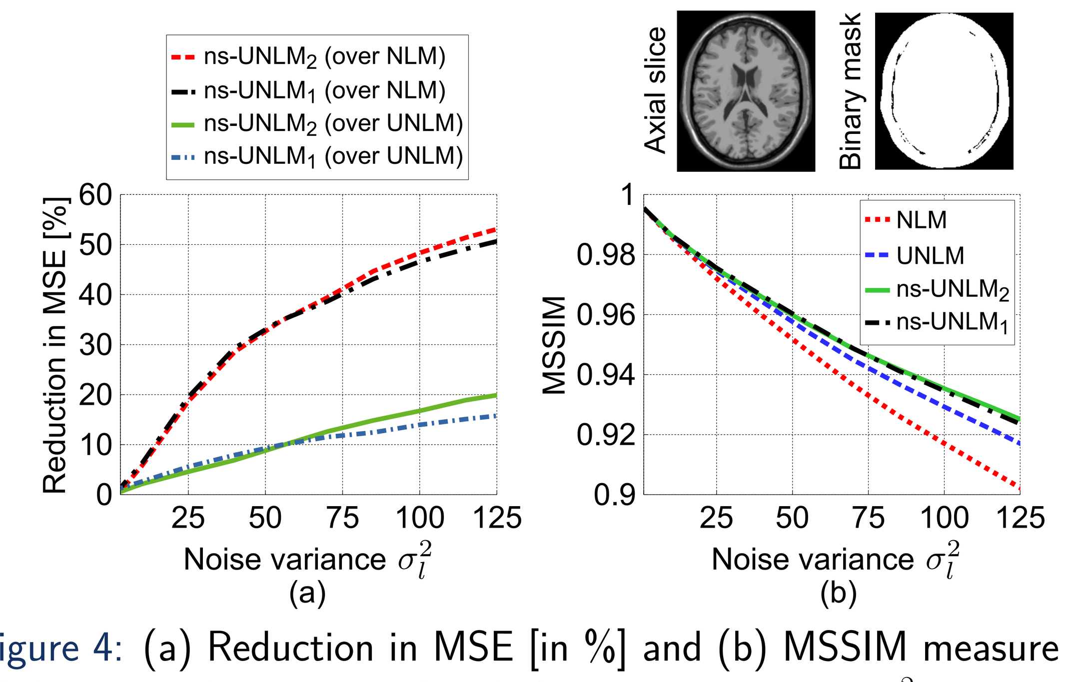

Figure 4: (a) Reduction in MSE [in %] and (b) MSSIM measure of the methods in terms of underlying noise variance σ_l^2.

Diffusion-weighted MR data

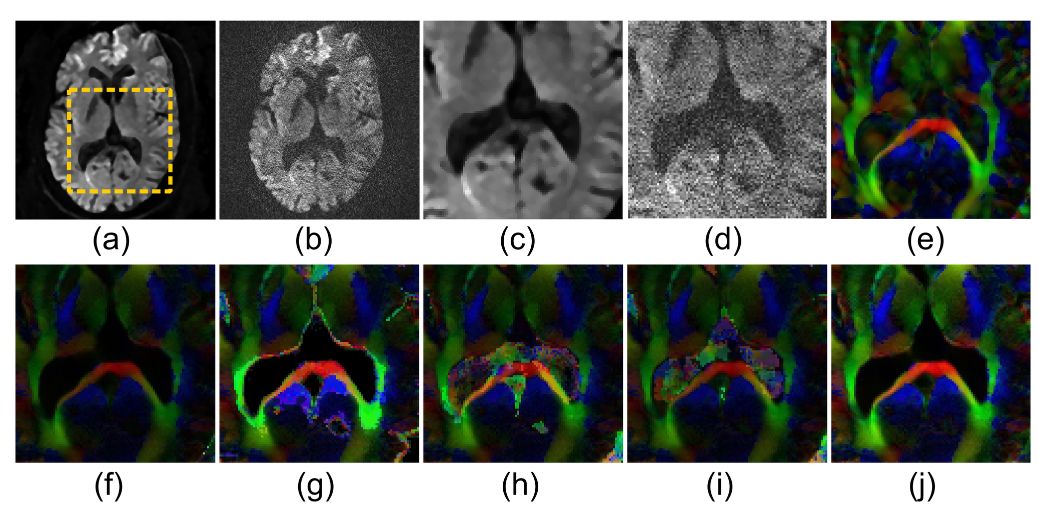

Figure 5: (a, c) Reference, (b, d) noisy and (e) FA map. FA obtained from denoised data: (f) NLM, (g) UNLM $L(j) = 2$, (h) UNLM $L(j) = 1$, (i) ns-UNLM$_2$ and (j) ns-UNLM$_1$.

T_1-weighted MR data

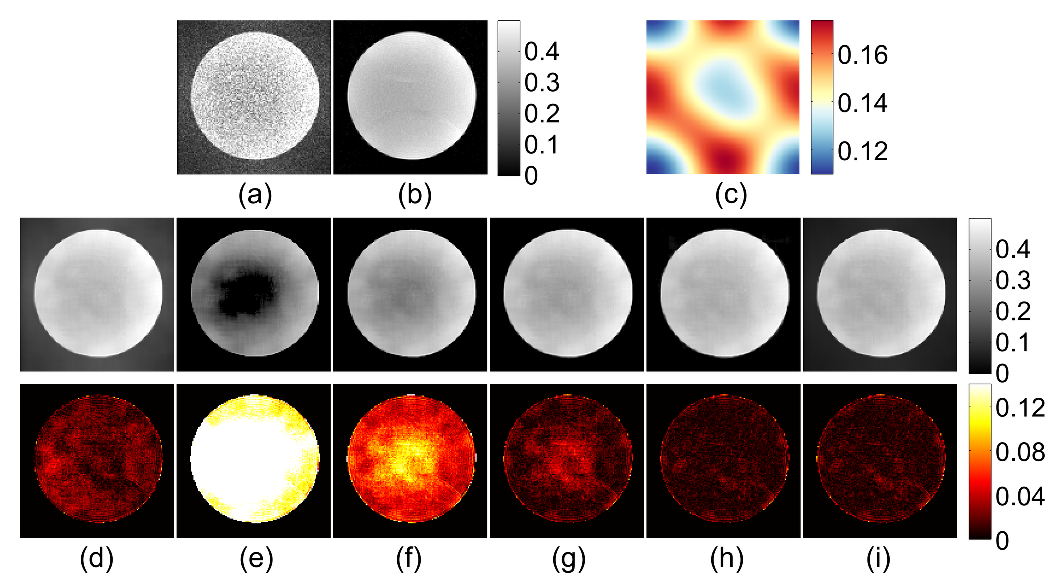

Figure 6: (a) Noisy, (b) p-UNLM$_2$ and (c) noise pattern.

Denoised: (d) NLM, (e) UNLM $L(j) = 8$, (f) UNLM $L(j) = 3$, (g) UNLM $L(j) = 2$, (h) ns-UNLM$_2$ and (i) ns-UNLM$_1$. Third row: absolute errors regarding (b).

The pseudoreference method p-UNLM$_2$ is calculated across $K = 100$ replicas.