Noise estimation and diffusion signal reconstruction: From cradle to parallel imaging

Santiago Aja-Fernández*

September 15, 2010

Abstract

Noise is known to be one of the main sources of quality deterioration in magnetic resonance (MR) data. Not only visual inspection but also processing techniques such as segmentation, registration or tensor estimation in diffusion tensor MRI (DT-MRI) will be affected or biased due to the presence of noise. In High Angular Resolution Diffusion Imaging (HARDI) for instance, to achieve a speedup in the acquisition time the temporal averaging is reduced; as a consequence, the strength of the noise is increased proportionally to the square root of the speedup. Noise in the k-space in MR data is usually assumed to be a zero-mean uncorrelated complex Gaussian process in each scanner coil, with equal variance in both the real and imaginary parts. As a result, sometimes it is easy to derive the distributiuon of the magnitude data (as in single coil systems) and other not-so easy, as in multiple coil parallel reconstruction.

Many filtering methods have been proposed either to remove the noise or to estimate the signal out of the noise. Different approaches may be found in the literature: (1) general filtering methods (anisotropic diffusion filtering, total variation methods) and (2) methods based on the statistical properties of the signal and noise (LMMSE, ML...). The talk will concentrate on discussing the nature of noise and means to address the challenges associated with it.

1 MRI Noise models

Noise in the **k**-space in Magnetic Resonance (MR) data from each coil is assumed to be a zero-mean uncorrelated Gaussian process with equal variance in both the real and imaginary parts. As a result, in single coil systems magnitude data in the spatial domain is modeled using a Rician distribution Drumheller (1993); Gudbjartsson and Patz (1995). In the same way, the composite signal in coils systems with multiple channels may be modeled as non–central Chi distributed Constantinides et al. (1997) if no subsampling of the **k**-space is assumed. The acquisition rate can be increased with parallel MRI (pMRI) techniques via subsampled acquisitions of the **k**-space data. In these cases, reconstruction methods have to be used in order to suppress the aliasing and underlying artifacts created by the subsampling. Dominant among these are SENSE Pruessmann et al. (1999) and GRAPPA Griswold et al. (2002), reviews of which can be found in Hoge et al. (2005); Larkman and Nunes (2007). From a statistical point of view, such a reconstruction will affect the stationarity of the noise in the reconstructed data, i.e. the spatial distribution of the noise across the image Thünberg and Zetterberg (2007). As a result, the variance of noise may vary for different image locations. Moreover it may also vary from one coil to another. However, under the assumption of a nearly homogeneous variance Dietrich et al. (2008), the data may be considered to follow a general non–central Chi distribution Aja-Fernández et al. (2010); Aja-Fernández et al. (2010a). This distribution reduces to a Rician if SENSE is used.

*S. Aja-Fernández is this the LPI, ETSI. Telecomunicacion, Universidad de Valladolid, Spain. sanaja@tel.uva.es. Special Thanks to A. Tristan-Vega and C.F. Westin. This work has been partly funded by Junta de Castilla y León with grants SAN126/VA33/09 and VA0339A10-2.

Main references

- Rician distribution: Drumheller (1993), Gudbjartsson and Patz (1995), Macovski (1996)
- Multiple-coil imaging models: Harpen (1992), Constantinides et al. (1997)
- Parallel imaging: Thünberg and Zetterberg (2007), Dietrich et al. (2008) (SENSE and GRAPPA, simulated), Aja-Fernández et al. (2010), Aja-Fernández et al. (2010a)
- Other studies: Aja-Fernández et al. (2010b).

2 Noise estimation

Noise in MR data, either from multiple or single coil acquisitions, is known to affect the visual quality of the MR images and different processing techniques, such as segmentation, registration or tensor estimation in Diffusion Tensor MRI (DT-MRI) Aja-Fernández et al. (2008b). Accordingly, the estimated noise power gives a measure of the quality of the data. This estimation can be used to measure the Signal-to-Noise Ratio (SNR) and as an input parameter in MR processing algorithms. Many filtering methods to *improve* SNR in MRI need an estimated value for σ_n^2: as the conventional approach McGibney and Smith (1993), maximum likelihood based methods Jiang and Yang (2003); Sijbers et al. (1998a,c), expectation maximization formulations with Rician noise assumptions DeVore et al. (2000), Linear Minimum Mean Square Error (LMMSE) based schemes Aja-Fernández et al. (2007, 2008a,b); Tristán-Vega and Aja-Fernández (2008) and unbiased non-local mean schemes Aja-Fernández and Krissian (2008); Coupé et al. (2008); Wiest-Daesslé et al. (2008). New techniques for DTI tensor estimation Fillard et al. (2007); Landman et al. (2007), segmentation methods based on the Rician distribution and fiber orientation estimators Clarke et al. (2008) also depend upon an estimated σ_n^2 value.

Noise estimation in MR is usually done over the (composite) magnitude image, since it is the usual output of the scanning process. However, if data in the complex spatial domain are available, the estimation may be easier done in that domain. Noise estimation is carried out assuming that the noise is uncorrelated and with identical variance in each pixel and it will be reduced to the estimation of the variance in a well-known Gaussian problem over one of the components.

If no complex data are available, noise estimation is carried out over the composite magnitude image, assuming again that the noise is uncorrelated and with identical variance in each pixel. Methods performing such estimation from magnitude data may roughly be divided into two groups: (i) approaches estimating the noise variance using a single magnitude image and (ii) approaches using multiple images. In this paper we will focus on the former. Noise estimation using a single image is usually based on background intensities, where the true signal amplitude should vanish and the Rayleigh or central Chi assumptions hold. Some of these techniques require a previous background segmentation. See overview in Table 2. Notation in Table 1.

Main references

- Gaussian noise estimation: Aja-Fernández et al. (2009b), Salmeri et al. (2001), Donoho and Johnstone (1994), Starck and Murtagh (1998)
- Rician noise estimation: Nowak (1999); Sijbers et al. (1998a,b,c) (background based) Aja-Fernández et al. (2009a); Brummer et al. (1993); Chang et al. (2005); Sijbers et al. (2006, 2007) (maximization of argument), Aja-Fernández et al. (2008a, 2009a); Sijbers et al. (2006) (mode based).
- Non-central χ: Aja-Fernández et al. (2009a); Constantinides et al. (1997); Dietrich et al. (2008); Koay and Basser (2006)

$E\{X\}$	Expectation of random variable X
σ_X^2	Variance of random variable X
$\langle M(\mathbf{x})\rangle$	(Global) Sample mean of image $M(\mathbf{x})$ $\langle M(\mathbf{x})\rangle = \frac{1}{\lvert\Omega\rvert}\sum\limits_{\mathbf{x}\in\Omega} M(\mathbf{x})$
$\langle M(\mathbf{x})\rangle_{\mathbf{x}}$	Local sample local mean of image $M(\mathbf{x})$ $\langle M(\mathbf{x})\rangle_{\mathbf{x}} = \frac{1}{\lvert\eta(\mathbf{x})\rvert}\sum\limits_{\mathbf{p}\in\eta(\mathbf{x})} M(\mathbf{p})$ ($\eta(\mathbf{x})$ a neighborhood centered in $\mathbf{x}$)
$\text{Var}(M(\mathbf{x}))_{\mathbf{x}}$	Sample local variance of $M(\mathbf{x})$ $\text{Var}(M(\mathbf{x}))_{\mathbf{x}} = \langle M^2(\mathbf{x})\rangle_{\mathbf{x}} - \langle M(\mathbf{x})\rangle_{\mathbf{x}}^2$
$M(\mathbf{x_B})$	Background area of image $M(\mathbf{x})$ $\mathbf{x_B} = \mathbf{x} \vert A(\mathbf{x}) = 0$
$M(\mathbf{x_R})$	$M(\mathbf{x})$ in the region $\mathbf{R}$ $\mathbf{x_R} \in \mathbf{R}$
$\text{mode}\{I(\mathbf{x})\}$	Mode of the distribution of $I(\mathbf{x})$ $\text{mode}\{I(\mathbf{x})\} = \arg\max_{\mathbf{x}}\{p_I(\mathbf{x})\}$
$\widehat{a}$	Estimator of parameter a

Table 1: Notation

3 Filtering and signal estimation

Since the amount of data to acquire in DTI studies is usually large, there is a tradeoff between acquisition time and quality that leads to a poorer Signal to Noise Ratio (SNR) associated to the Diffusion Weighted Images (DWI) used to infer fiber orientations when compared to conventional Magnetic Resonance Imaging (MRI). This is especially the case with modern High Angular Resolution Diffusion Imaging (HARDI) techniques, where very strong gradients have to be applied in order to improve the angular contrast. This produces a strong attenuation which worsens the SNR. Moreover, the Rician nature of the noise in DWI prevents the use of conventional Gaussian-based filtering techniques.

Main references

- Anisotropic Diffusion Schemes: Gerig et al. (1992); Krissian and Aja-Fernández (2009); Perona and Malik (1990); Weickert et al. (1998)

- Rician Based: McGibney and Smith (1993) (Conventional approach), Basu et al. (2006); Koay and Basser (2006),

- Bayesian Framework: Marzetta (1995) (Expectation Maximization, EM, estimation of Rician signal)Sijbers et al. (1998a); Sijbers and den Dekker (2004); Sijbers et al. (1998c) (maximum likelihood, ML, signal estimation), Aja-Fernández et al. (2007, 2008a,b); Martin-Fernandez et al. (2007); Tristán-Vega and Aja-Fernández (2008) (Linear Minimum Mean Squared Error, LMMSE, signal estimation), Tristán-Vega and Aja-Fernández (2008, 2010) (LMMSE and NLM taking into account information of all DWIs).

- Nonlocal statistics: Buades et al. (2005); Coupé et al. (2008) (Nonlocal mean), Descotaux et al. (2008); Xu et al. (2008) (bias in NLM) Aja-Fernández and Krissian (2008); Manjón et al. (2008); Wiest-Daesslé et al. (2008) (NLM with Rician correction).

- Other methods: Awate and Whitaker (2005) (Nonparametric Neighborhood Statistics), Jiang and Yang (2003), Nowak (1999); Pižurica et al. (2003) (Wavelets), Ahn et al. (1999); McGraw et al. (2004)

	Assump.	Method	Refs.
1	Rayleigh	$\widehat{\sigma_n^2} = \frac{1}{2}\langle M^2(\mathbf{x_B})\rangle$	Sijbers et al. (1998b)
	Rayleigh	$\widehat{\sigma_n} = \sqrt{\frac{2}{\pi}}\langle M(\mathbf{x_B})\rangle$	Sijbers et al. (1998b)
2-a	Rayleigh	$\widehat{\sigma_n} = \text{mode}\{M(\mathbf{x})\}$	Brummer et al. (1993); Sijbers et al. (2006)
	Rayleigh	$\widehat{\sigma_n^2} = \frac{1}{2}\text{mode}\{\langle M^2(\mathbf{x})\rangle_{\mathbf{x}}\}$	Aja-Fernández et al. (2008a)
	Rayleigh	$\widehat{\sigma_n} = \sqrt{\frac{N}{2N-1}}\text{mode}\left\{\sqrt{\langle M^2(\mathbf{x})\rangle_{\mathbf{x}}}\right\}$	Aja-Fernández et al. (2009a)
	Rayleigh	$\widehat{\sigma_n} = \sqrt{\frac{2}{\pi}}\text{mode}\{\langle M(\mathbf{x})\rangle_{\mathbf{x}}\}$	Aja-Fernández et al. (2008a)
	Rayleigh	$\widehat{\sigma_n^2} = \frac{2}{4-\pi}\text{mode}\{\text{Var}(M(\mathbf{x}))_{\mathbf{x}}\}$	Aja-Fernández et al. (2008a)
2-b	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{l=l_0}^{l_c}\left(h_M(l) - K\frac{l}{\sigma^2}e^{-\frac{l^2}{2\sigma^2}}\right)^2$	Brummer et al. (1993)
	Rayleigh	$\widehat{\sigma_n} = \arg\max_{\sigma}\frac{1}{nh}\sum_{i=0}^{n}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\sigma-x_i}{h}\right)^2}$	Chang et al. (2005)
	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{l=l_0}^{l_c}\left(h_{<M^2>}(l) - K\frac{l^{N-1}N^N}{(2\sigma^2)^N\Gamma(N)}e^{-\frac{lN}{2\sigma^2}}\right)^2$	Aja-Fernández et al. (2009a)
	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{l=l_0}^{l_c}\left(h_{<M>}(l) - K\frac{l^{2N-1}N^N}{2^{N-1}b^N\Gamma(N)}e^{-\frac{l^2N}{2b}}\right)^2$	Aja-Fernández et al. (2009a)
	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{l=l_0}^{l_c}\left(h_{\sqrt{<M^2>}}(l) - K\frac{l^{2N-1}N^N}{2^{N-1}\sigma^{2N}\Gamma(N)}e^{-\frac{l^2N}{2\sigma^2}}\right)^2$	Aja-Fernández et al. (2009a)
2-c	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma}\left[N_k\log\left(e^{-\frac{l_0^2}{2\sigma^2}} - e^{-\frac{l_k^2}{2\sigma^2}}\right) - \sum_{i=1}^{k} n_i\log\left(e^{-\frac{l_{i-1}^2}{2\sigma^2}} - e^{-\frac{l_i^2}{2\sigma^2}}\right)\right]$	Sijbers et al. (2007)
	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma}\left[N_k\log\left(\Gamma\left(N+1, l_0\frac{N}{2\sigma^2}\right) - \Gamma\left(N+1, l_k\frac{N}{2\sigma^2}\right)\right) - \sum_{i=1}^{k} n_i\log\left(\Gamma\left(N+1, l_{i-1}\frac{N}{2\sigma^2}\right) - \Gamma\left(N+1, l_i\frac{N}{2\sigma^2}\right)\right)\right]$	Aja-Fernández et al. (2009a)
	Rayleigh	$\widehat{\sigma_n} = \arg\min_{\sigma}\left[N_k\log\left(\Gamma\left(N, \frac{l_0^2N}{2\sigma^2}\right) - \Gamma\left(N, \frac{l_k^2N}{2\sigma^2}\right)\right) - \sum_{i=1}^{k} n_i\log\left(\Gamma\left(N, \frac{l_{i-1}^2N}{2\sigma^2}\right) - \Gamma\left(N, \frac{l_i^2N}{2\sigma^2}\right)\right)\right]$	Aja-Fernández et al. (2009a)
3	Rician	$\widehat{\sigma_n^2} = \text{mode}\{\text{Var}(M(\mathbf{x}))_{\mathbf{x}}\}$	Aja-Fernández et al. (2008a)
4	central χ	$\widehat{\sigma_n^2}_L = \frac{1}{2}\langle M_L^2(\mathbf{x_B})\rangle$	Constantinides et al. (1997)
	central χ	$\widehat{\sigma_n}_L = \frac{1}{\sqrt{2}}\langle M_L(\mathbf{x_B})\rangle\frac{\sqrt{L}\Gamma(L)}{\Gamma(L+\frac{1}{2})}$	Dietrich et al. (2008)
5-a	central χ	$\widehat{\sigma_n} = \frac{1}{\sqrt{2L-1}}\text{mode}\{M_L(\mathbf{x})\}$	Aja-Fernández et al. (2009a)
	central χ	$\widehat{\sigma_n^2}_L = \frac{1}{2}\text{mode}\{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}\}$	Aja-Fernández et al. (2009a)
	central χ	$\widehat{\sigma_n}_L = \frac{1}{\sqrt{2}}\text{mode}\{\langle M_L(\mathbf{x})\rangle_{\mathbf{x}}\}\frac{\sqrt{L}\Gamma(L)}{\Gamma(L+\frac{1}{2})}$	Aja-Fernández et al. (2009a)
	central χ	$\widehat{\sigma_n^2} = \left(2L - \frac{2\Gamma^2(L+\frac{1}{2})}{\Gamma^2(L)}\right)^{-1}\text{mode}\{\text{Var}(M_L(\mathbf{x}))_{\mathbf{x}}\}$	Aja-Fernández et al. (2009a)
5-b	central χ	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{m=m_0}^{m_c}\left(h_M(m) - K\frac{2^{1-L}}{\Gamma(L)}\frac{m^{2L-1}}{\sigma^{2L}}e^{-\frac{m^2}{2\sigma^2}}\right)^2$	Aja-Fernández et al. (2009a)
	central χ	$\widehat{\sigma_n} = \arg\min_{\sigma,K}\sum_{m=m_0}^{m_c}\left(h_{<M_L^2>}(l) - K\frac{m^{NL-1}N^{NL}}{(2\sigma^2)^{NL}\Gamma(NL)}e^{-\frac{mN}{2\sigma^2}}\right)^2$	Aja-Fernández et al. (2009a)
5-c	central χ	$\widehat{\sigma_n} = \arg\min_{\sigma}\left[N_k\log\left(\Gamma\left(L, \frac{m_0^2}{2\sigma^2}\right) - \Gamma\left(L, \frac{m_c^2}{2\sigma^2}\right)\right) - \sum_{i=1}^{K} n_i\log\left(\Gamma\left(L, \frac{m_{i-1}^2}{2\sigma^2}\right) - \Gamma\left(L, \frac{m_i^2}{2\sigma^2}\right)\right)\right]$	Aja-Fernández et al. (2009a)
	central χ	$\widehat{\sigma_n} = \arg\min_{\sigma}\left[N_k\log\left(\Gamma\left(NL, m_0\frac{N}{2\sigma^2}\right) - \Gamma\left(NL, m_k\frac{N}{2\sigma^2}\right)\right) - \sum_{i=1}^{K} n_i\log\left(\Gamma\left(NL, m_{i-1}\frac{N}{2\sigma^2}\right) - \Gamma\left(NL, m_i\frac{N}{2\sigma^2}\right)\right)\right]$	Aja-Fernández et al. (2009a)
6	non-central χ	$\widehat{\sigma_n^2} = \text{mode}\{\text{Var}(M_l(\mathbf{x}))_{\mathbf{x}}\}$	Aja-Fernández et al. (2009a)
7	Gaussian	$\widehat{\sigma_n^2} = \text{mode}\{\text{Var}(C_{l_j}(\mathbf{x}))_{\mathbf{x}}\}$	Aja-Fernández et al. (2009b)

Table 2: Survey of noise estimators for single and multiple coil MR data. Note that estimators in boxes 1 and 4 require background segmentation. From Aja-Fernández et al. (2009a)

- Phantom for DWI filtering: Tristán-Vega and Aja-Fernández (2009, 2010)

4 Noise and tensor estimation

Despite the known weakness of the single tensor approach in certain cases, as fiber crossing or kissing, Diffusion Tensor Imaging (DTI) is still widely used, mainly due to its simplicity and straight visual interpretation. Least Squares (LS) techniques have become the *de facto* standard to estimate the Diffusion Tensor (DT) in DTI Salvador et al. (2005); Tristán-Vega et al. (2009). Although other approaches are possible, LS are fast and robust, and they show optimal properties when Weighted Least Squares (WLS) is used. Recent studies show that when single coil acquisition is considered, the estimation is nearly unbiased, so WLS is in this case the Best Linear Unbiased Estimator (BLUE) Salvador et al. (2005). When multiple receiver coils for simultaneous acquisition are considered, the bias may be an important source of error; the larger the number of receiving coils the more critical. As stated in Tristán-Vega et al. (2009), the variance in the estimation may be reduced by increasing the number of gradient directions, but the error bias remains. Results in Salvador et al. (2005); Tristán-Vega et al. (2009) provide a theoretical framework to properly estimate the DT from single- and multiple-coil systems.

Main references

- Salvador et al. (2005) (WLS assuming Rician distribution), Landman et al. (2007) (ML assuming Rician distribution), Tristán-Vega et al. (2009) (bias of WLS estimation for nc-χ distribution), Aja-Fernández et al. (2010) (simulations for parallel imaging).

5 Image quality assessment methods

Full-reference methods for quality assessment are those in which a signal is compared to a *ground truth* image, i.e. a *golden standard*. Within these methods, the most frequently used are those error based methods, as the Mean Squared Error (MSE) Eskicioglu and Fisher (1995); Tang and Cahill (1992). The limitations of such methods have been widely reported in the literature (see Girod (1993) for example). Consequently, some additional variations of the MSE have also been used in order to better deal with the features of the Human Visual System Eskicioglu and Fisher (1995); Tang and Cahill (1992). In Miyahara et al. (1998) a new index is proposed, namely, the objective Picture Quality Scale (PQS), basically intended to measure the degradation in coding and compression of images. It takes into account properties of visual perception of both global features and of disturbances. It turns out to be bounded, being the maximum value 5.797, obtained when an image is compared with itself. Experiments show that although it is a good measure when dealing with compression, it is not so good a measure for other sources of degradation Sheikh et al. (2006). Recently, some methods based on *Natural Scene Statistics* have been reported Sheikh and Bovik (2006); Sheikh et al. (2005).

In Wang et al. (2004) Wang *et al.* proposed a full-reference quality assessment method based on the structural similarity of two images, the so-called Structural Similarity (SSIM) index. The method is a modification of their Quality Index, originally proposed in Wang and Bovik (2002). As of today, this method has proved to be versatile and robust in many different environments Sheikh et al. (2006). In Aja-Fernández et al. (2006) a new method based on the structural information of the image and, specifically, on the statistics of the sample local variance; it is intended to penalize degradations in which the structural content is filtered out by accounting for the non-stationary content in the image. Additionally, it intends not to over-penalize noise as long as noise content does not obscure structure.

Main references

- Phantoms: Collins et al. (1998) (BrainWeb) and Tristán-Vega and Aja-Fernández (2009, 2010) (DWI phantom).

- Full-reference methods: Wang et al. (2004) (SSIM), Aja-Fernández et al. (2006) (QILV), Eskicioglu and Fisher (1995); Tang and Cahill (1992) (MSE).

References

Ahn, C. B., Song, Y. C., Park, D. J., 1999. Adaptive template filtering for signal-to-noise ratio enhancement in magnetic resonance imaging. IEEE Trans. Med. Imaging 18 (6), 549–556.

Aja-Fernández, S., Alberola-López, C., Westin, C.-F., 2007. Signal LMMSE estimation from multiple samples in MRI and DT-MRI. In: Proceedings of MICCAI 07. Lecture Notes on Computer Science. Vol. 4792. pp. 368–375.

Aja-Fernández, S., Alberola-López, C., Westin, C.-F., Aug. 2008a. Noise and signal estimation in magnitude MRI and Rician distributed images: A LMMSE approach. IEEE Trans. Image Proccess. 17 (8), 1383–1398.

Aja-Fernández, S., Krissian, K., 2008. An unbiased non-local means scheme for dwi filtering. In: Proc. of the Workshop on Computational Diffusion MRI, MICCAI 2008. pp. 277–284.

Aja-Fernández, S., Niethammer, M., Kubicki, M., Shenton, M. E., Westin, C.-F., Oct. 2008b. Restoration of DWI data using a Rician LMMSE estimator. IEEE Trans. Med. Imaging 27 (10), 1389–1403.

Aja-Fernández, S., San-José-Estépar, R., Alberola-López, C., Westin, C.-F., Sep. 2006. Image quality assesment based on local variance. In: Proc of the 28th IEEE International Conference of the Engineering in Medicine and Biology Society (EMBC). New York, pp. 4815–4818.

Aja-Fernández, S., Tristán-Vega, A., Alberola-López, C., 2009a. Noise estimation in single and multiple coil MR data based on statistical models. Magn. Reson. Imag. 27, 1397–1409.

Aja-Fernández, S., Tristán-Vega, A., de-la Higuera, P. C., 2010. DWI acquisition schemes and diffusion tensor estimation: A simulationbased study. In: Proc. of the 32nd Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC'2010). Buenos Aires, Argentina.

Aja-Fernández, S., Tristán-Vega, A., Hoge, S., 2010. Statistical noise analysis in GRAPPA using a parametrized non-central chi approximation model. Magn. Reson. Med.Under second review.

Aja-Fernández, S., Tristán-Vega, A., Hoge, S., 2010a. Statistical noise model in GRAPPA-reconstructed images. In: Proc. of the ISMRM 2010. Stockholm, Sweden.

Aja-Fernández, S., Vegas-Sánchez-Ferrero, G., Martín-Fernández, M., Alberola-López, C., May 2009b. Automatic noise estimation in images using local statistics. Additive and multiplicative cases. Image and Vision Computing 27 (6), 756–770.

Aja-Fernández, S., Vegas-Sanchez-Ferrero, G., Tristán-Vega, A., 2010b. About the background distribution in MR data: a local variance study. Magn. Reson. Imag. 28, 739–752.

Awate, S., Whitaker, R., Jul. 2005. Nonparametric neighborhood statistics for MRI denoising. In: Information Processing in Medical Imaging. Vol. 3565 of Lecture Notes in Computer Science. Springer, pp. 677–688.

Basu, S., Fletcher, T., Whitaker, R., 2006. Rician noise removal in diffusion tensor MRI. In: Proceedings of MICCAI. Vol. 1. pp. 117–125.

Brummer, M., Mersereau, R., Eisner, R., Lewine, R., 1993. Automatic detection of brain contours in MRI data sets. IEEE Trans. Med. Imaging 12 (2), 153–166.

Buades, A., Coll, B., Morel, J., 2005. A review of image denoising algorithms, with a new one. Multiscale Modeling and Simulation 4 (2), 490–530.

Chang, L.-C., Rohde, G. K., Pierpaoli, C., 2005. An automatic method for estimating noise-induced signal variance in magnitude-reconstructed magnetic resonance images. Medical Imaging 2005: Image Processing 5747 (1), 1136–1142.

Clarke, R., Scifo, P., Rizzo, G., Dell'Acqua, F., Scotti, G., Fazio, F., Sep. 2008. Noise correction on Rician distributed data for fibre orientation estimators. IEEE Trans. Med. Imaging 27 (9), 1242–1251.

Collins, D., Zijdenbos, A., Killokian, V., Sled, J., Kabani, N., Holmes, C., Evans, A., Jun. 1998. Design and construction of a realistic digital brain phantom. IEEE Trans. Med. Imaging 17 (3), 463–468.

Constantinides, C., Atalar, E., McVeigh, E., 1997. Signal-to-noise measurements in magnitude images from NMR based arrays. Magn. Reson. Med. 38, 852–857.

Coupé, P., Yger, P., Prima, S., Hellier, P., Kervrann, C., C.Barillot, Apr. 2008. An optimized blockwise non local means denoising filter for 3D magnetic resonance images. IEEE Trans. Med. Imaging 27, 425–441.

Descotaux, M., Wiest-Daesslé, N., Prima, S., Barillot, C., Deriche, R., 2008. Ipmact of Rician adapted Non–Local Means filtering on HARDI. In: Medical Image Computing and Computer-Assisted Intervention. Vol. 5242 of Lecure Notes in Computer Science. Springer–Verlag, pp. 122–130.

DeVore, M. D., Lanterman, A. D., O'Sullivan, J. A., Apr. 2000. ATR performance of a Rician model for SAR images. In: Proc. of SPIE 2000. Orlando, pp. 34–37.

Dietrich, O., Raya, J., Reeder, S. B., Ingrisch, M., Reiser, M., Schoenberg, S. O., 2008. Influence of multichannel combination, parallel imaging and other reconstruction techniques on MRI noise characteristics. Magn. Reson. Imag. 26, 754–762.

Donoho, D., Johnstone, I., Apr. 1994. Ideal spatial adaptation by wavelet shrinkage. Biometrika 81, 425–455.

Drumheller, D., Apr. 1993. General expressions for Rician density and distribution functions. IEEE Trans. on Aerospace and Electronic Systems 29 (2), 580–588.

Eskicioglu, A., Fisher, P., Dec. 1995. Image quality measures and their performance. IEEE Tr. Comm. 43 (12), 2959–2965.

Fillard, P., Pennec, X., Arsigny, V., Ayache, N., Nov. 2007. Clinical DT-MRI estimation, smoothing, and fiber tracking with log-Euclidean metrics. IEEE Trans. Med. Imaging 26 (11), 1472–1482.

Gerig, G., Kübler, O., Kikinis, R., Jolesz, F., Jun. 1992. Nonlinear anisotropic filtering of MRI data. IEEE Trans. Med. Imaging 11 (2), 221–232.

Girod, B., 1993. Digital images and human vision. MIT Press, Cambridge, MA, USA, Ch. What's wrong with mean-squared error?, pp. 207–220.

Griswold, M. A., Jakob, P. M., Heidemann, R. M., Nittka, M., Jellus, V., Wang, J., Kiefer, B., Haase, A., 2002. Generalized autocalibrating partially parallel acquisitions (GRAPPA). Magn. Reson. Med. 47 (6), 1202–1210.

Gudbjartsson, H., Patz, S., 1995. The Rician distribution of noisy MRI data. Magn. Reson. Med. 34 (6), 910–914.

Harpen, M., 1992. Noise correlations exist for independent RF coils. Magn. Reson. Med. 23, 394–397.

Hoge, W. S., Brooks, D. H., Madore, B., Kyriakos, W. E., Sep. 2005. A tour of accelerated parallel MR imaging from a linear systems perspective. Concepts Magn Reson Part A 27A (1), 17–37.

Jiang, L., Yang, W., Dec. 2003. Adaptive magnetic resonance image denoising using mixture model and wavelet shrinkage. In: Proc. VIIth digital image computing: techniques and applications. Sydney, Australia, pp. 831–838.

Koay, C. G., Basser, P. J., 2006. Analytically exact correction scheme for signal extraction from noisy magnitude MR signals. J. Magn. Reson. 179, 317–322.

Krissian, K., Aja-Fernández, S., 2009. Noise-driven anisotropic diffusion filtering of mri. IEEE Trans. Image Proccess. 18 (10), 2265–2274.

Landman, B., Bazin, P.-L., Prince, J., Oct. 2007. Diffusion tensor estimation by maximizing Rician likelihood. In: Proceedings of the ICCV 2007. pp. 1–8.

Larkman, D. J., Nunes, R. G., 2007. Parallel magnetic resonance imaging. Physics in Medicine and Biology 52, 15–55, invited Topical Review.

Macovski, A., 1996. Noise in MRI. Magn. Reson. Med. 36, 494–497.

Manjón, J. V., Carbonell-Caballero, J., Lull, J. J., García-Martí, G., Martí-Bonmatí, L., Robles, M., 2008. MRI denoising using Non-Local Means. Med. Imag. Anal. 12, 514–523.

Martin-Fernandez, M., Aberola-Lopez, C., Ruiz-Alzola, J., Westin, C.-F., 2007. Sequential anisotropic Wiener filtering applied to 3D MRI data. Magn. Reson. Imag. 25, 278–292.

Marzetta, T., 1995. EM algorithm for estimating the parameters of multivariate complex Rician density for polarimetric SAR. In: Proc. of ICASSP. Vol. 5. pp. 3651–3654.

McGibney, G., Smith, M., 1993. Unbiased signal-to-noise ratio measure for magnetic resonance images. Med. Phys. 20 (4), 1077–1078.

McGraw, T., Vemuri, B. C., Chen, Y., Rao, M., Mareci, T., 2004. DT-MRI denoising and neuronal fiber tracking. Med. Imag. Anal. 8, 95–111.

Miyahara, M., Kotani, K., Algazi, V. R., Sep. 1998. Objective picture quality scale (PQS) for image coding. IEEE Tr. Comm. 46 (9), 1215–1226.

Nowak, R., Oct. 1999. Wavelet-based Rician noise removal for Magnetic Resonance Imaging. IEEE Trans. Image Proccess. 8 (10), 1408–1419.

Perona, P., Malik, J., Jul. 1990. Scale-space and edge detection using anisotropic diffusion. IEEE Trans. Pattern Anal. Mach. Intell. 12 (7), 629–639.

Pižurica, A., Philips, W., Lemahieu, I., Acheroy, M., Mar. 2003. A versatile Wavelet domain noise filtration technique for medical imaging. IEEE Trans. Med. Imaging 22 (3), 323–331.

Pruessmann, K. P., Weiger, M., Scheidegger, M. B., Boesiger, P., Nov. 1999. SENSE: Sensitivity encoding for fast MRI. Magn. Reson. Med. 42 (5), 952–62.

Salmeri, M., Mencattini, A., Ricci, E., Salsano, A., Oct. 2001. Noise estimation in digital images using fuzzy processing. In: Proc. of the IEEE Int.l Conf. on Image Processing. Thessaloniki (Greece), pp. 517–520.

Salvador, R., Peña, A., Menon, D. K., Carpenter, T., Pickard, J., Bullmore, E. T., Feb. 2005. Formal characterization and extension of the linearized diffusion tensor model. Human brain mapping 24, 144–155.

Sheikh, H., Bovik, A., Feb. 2006. Image information and visual quality. IEEE Trans. Image Proccess. 15 (2), 430–444.

Sheikh, H., Bovik, A. C., de Veciana, G., Dec. 2005. An information fidelity criterion for image quality assessment using natural scene statistics. IEEE Trans. Image Proccess. 14 (12), 2117–2128.

Sheikh, H., Sabir, M., Bovik, A., Nov. 2006. A statistical evaluation of recent full reference image quality assessment algorithms. IEEE Trans. Image Proccess. 15 (11), 3440–3451.

Sijbers, J., den Dekker, A., Scheunders, P., Van Dyck, D., Jun. 1998a. Maximum-likelihood estimation of Rician distribution parameters. IEEE Trans. Med. Imaging 17 (3), 357–361.

Sijbers, J., den Dekker, A., Van Audekerke, J., Verhoye, M., Van Dyck, D., 1998b. Estimation of the noise in magnitude MR images. Magn. Reson. Imag. 16 (1), 87–90.

Sijbers, J., den Dekker, A. J., 2004. Maximum likelihood estimation of signal amplitude and noise variance form MR data. Magn. Reson. Imag. 51, 586–594.

Sijbers, J., den Dekker, A. J., Poot, D., Verhoye, M., Van Camp, N., Van der Linden, A., 2006. Robust estimation of the noise variance from background MR data. In: Proc. of SPIE. Medical Imaging 2006: Image Processing. Vol. 6144. pp. 2018–2028.

Sijbers, J., den Dekker, A. J., Van Dyck, D., Raman, E., Feb. 1998c. Estimation of signal and noise from Rician distributed data. In: Proc. of the Int. Conf. on Signal Proc. and Comm. Las Palmas de Gran Canaria, Spain, pp. 140–142.

Sijbers, J., Poot, D., den Dekker, A. J., Pintjenst, W., Feb. 2007. Automatic estimation of the noise variance from the histogram of a magnetic resonance image. In: Physics in Medicine and Biology. Vol. 52. pp. 1335–1348.

Starck, J.-L., Murtagh, F., Feb. 1998. Automatic noise estimation from the multiresolution support. Publications of the Astronomical Society of the Pacific 110, 193–199.

Tang, H., Cahill, L., Nov. 1992. A new criterion for the evaluation of image restoration quality. In: Proc. of the IEEE Region 10 Conf. Tencon 92. Melbourne, Australia, pp. 573–577.

Thünberg, P., Zetterberg, P., 2007. Noise distribution in SENSE- and GRAPPA- reconstructed images: a computer simulation study. Magn. Reson. Imag. 25, 1089–94.

Tristán-Vega, A., Aja-Fernández, S., 2008. Joint LMMSE estimation of DWI data for DTI processing. In: Proceedings of MICCAI 08. Lecture Notes on Computer Science. Vol. 5242. pp. 27–34.

Tristán-Vega, A., Aja-Fernández, S., 2009. Design and construction of a realistic dwi phantom for filtering performance assessment. In: Yang, G.-Z., Hawkes, D., Rueckert, D., Noble, A., Taylor, C. (Eds.), Medical Image Computing and Computer-Assisted Intervention MICCAI 2009. Vol. 5761 of Lecture Notes in Computer Science. Springer Berlin / Heidelberg, pp. 951–958.

Tristán-Vega, A., Aja-Fernández, S., 2010. Dwi filtering using joint information for dti and hardi. Med. Imag. Anal. 14 (2), 205 – 218.

Tristán-Vega, A., Westin, C.-F., Aja-Fernández, S., 2009. Bias of least squares approaches for diffusion tensor estimation from array coils in DT–MRI. In: MICCAI 2009. Vol. 5761 of Lecture Notes in Computer Science. pp. 919–926.

Wang, Z., Bovik, A. C., Mar. 2002. A universal image quality index. IEEE Signal Processing Letters 9 (3), 81–84.

Wang, Z., Bovik, A. C., Sheikh, H. R., Simoncelli, E. P., Apr. 2004. Image quality assessment: from error visibility to structural similarity. IEEE Trans. Image Proccess. 13 (4), 600–612.

Weickert, J., ter Haar Romeny, B., Viergever, M. A., Mar. 1998. Efficient and reliable schemes for nonlinear diffusion filtering. IEEE Trans. Image Proccess. 7 (3), 398–410.

Wiest-Daesslé, N., Prima, S., Coupé, P., Morrissey, S., Barillot, C., 2008. Rician noise removal by non-local means filtering for low signal-to-noise ratio MRI: Applications to DT-MRI. In: Lecture Notes in Computer Science, MICCAI 2008. pp. 171–179.

Xu, H., Xu, J., Wu, F., 2008. On the biased estimation of Nonlocal Means filter. In: Proceedings of IEEE International Conference on Multimedia and Expo. pp. 1149–1152.