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Single-shell return-to-the-origin probability diffusion MRI measure under a non-stationary Rician distributed noise

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In a nutshell

Problem:

- The estimation of the Ensemble Average Propagator (EAP) and its related features such as the Return-To-the-Origin Probability (RTOP) measure requires a huge amount of densely sampled multiple-shell $\mathbf{q}$-space data.

Solution:

- We analytically derive an alternative approach to retrieve the RTOP directly from a single-shell $\mathbf{q}$-space data.
- We provide a closed-form solution to correct noise-induced bias using a non-stationary log-Rician statistics.

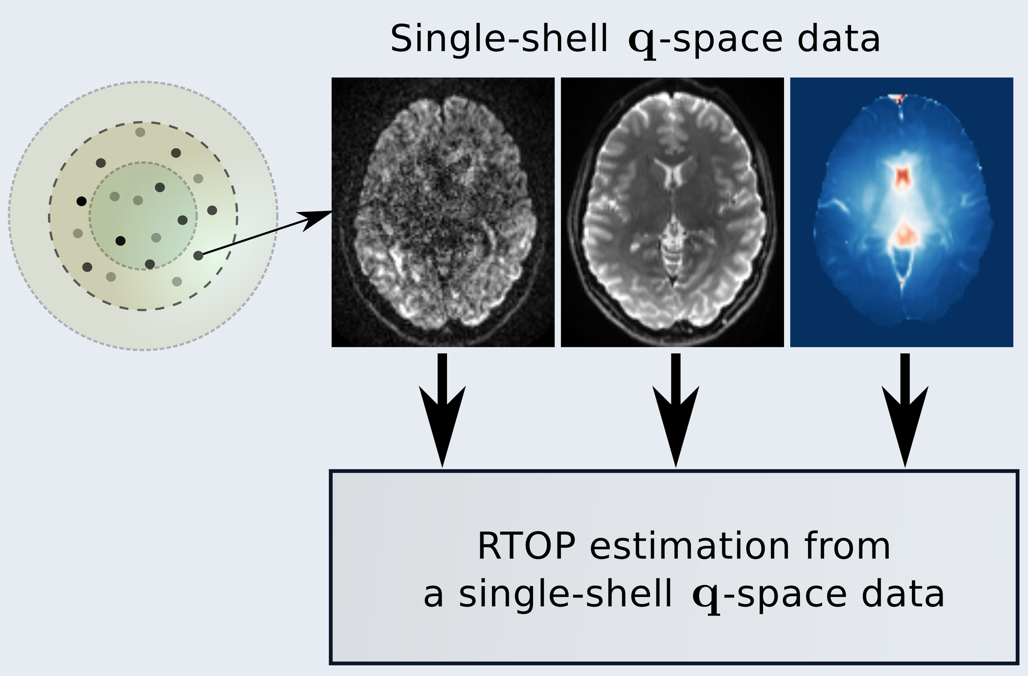

Figure 1: The RTOP estimation procedure.

Ensemble Average Propagator

Under the narrow pulse assumption, the EAP in real space, $P(\mathbf{R})$, is related to the diffusion signal attenuation $E(\mathbf{q})$ in the $\mathbf{q}$-space domain by means of the Fourier transform

$$P(\mathbf{R}) = \int_{\mathbb{R}^3} E(\mathbf{q}) \exp(-2\pi j \mathbf{q}^T \mathbf{R}) d\mathbf{q},$$

- $S(\mathbf{q})$ is the diffusion signal acquired at position $\mathbf{q}$,
- S_0 is the baseline measured without a diffusion sensitization,
- $\mathbf{q}$ is the wave vector related to $b = 4\pi^2\tau\|\mathbf{q}\|^2$ with τ being the effective diffusion time.

Return-To-the-Origin Probability

The probability in the origin indicates the EAP feature that the molecules minimally diffuse within the diffusion time and it is referred to as the RTOP measure

$$\mathrm{RTOP} = \int_{\mathbb{R}^3} E(\mathbf{q}) d\mathbf{q}.$$

Considering a more general model beyond the diffusion tensor, i.e. $E(\mathbf{q}) = \exp(-bD(\mathbf{q}))$, and assuming that the diffusion does not depend on the radial coordinate we can define the RTOP integral in a spherical system

$$\mathrm{RTOP} = \frac{\sqrt{\pi}}{4(4\pi^2\tau)^{3/2}} \int_0^{2\pi} \int_0^{\pi} (D(\theta, \phi))^{-3/2} \sin\theta \, d\theta \, d\phi,$$

- $D(\theta, \phi)$ is the apparent diffusion coefficient.

Numerical integration

In order to numerically evaluate the RTOP integral, one can use a direct approach assuming that the element of the surface, ΔS, is inversely proportional to the number of gradients (i.e. $\Delta S \propto 1/N_g$)

$$\mathrm{RTOP}^{(1)}(\mathbf{x}) = C_\tau \frac{1}{N_g} \sum_{i=1}^{N_g} \left(-\frac{1}{b} \log E(\mathbf{q}_i) \right)^{-3/2} = C_\tau b^{3/2} \left\langle (-\log E(\mathbf{q}_i))^{-3/2} \right\rangle,$$

- $C_\tau = 8^{-1}(\pi\tau)^{-3/2}$ is a time-related constant.

Considering the second-order Taylor expansion of the expectation operator $\mathbb{E}\{f(X)\}$ given $f(X) = X^{-3/2}$ we obtain the approximation

$$\mathbb{E}\left\{X^{-3/2}\right\} \approx \frac{1}{\mathbb{E}\{X\}^{3/2}} \left(\frac{15}{8} \frac{\mathbb{E}\{X^2\}}{\mathbb{E}\{X\}^2} - \frac{7}{8} \right),$$

and finally redefine direct $\mathrm{RTOP}^{(1)}$ formulation using a sample mean estimator $\mathbb{E}\{X^p\} = \langle (-\log E(\mathbf{q}_i))^p \rangle$

New solution

$$\mathrm{RTOP}^{(2)}(\mathbf{x}) = \frac{15}{8} C_\tau b^{3/2} \frac{\left\langle (\log E(\mathbf{q}_i))^2 \right\rangle}{\langle -\log E(\mathbf{q}_i) \rangle^{7/2}} - \frac{7}{8} C_\tau b^{3/2} \langle -\log E(\mathbf{q}_i) \rangle^{-3/2}.$$

Non-stationary log-Rician bias

Let us presume that the random variable $\log S_i(\mathbf{x})$ follows a non-stationary log-Rician distribution with the underlying parameters $A_i(\mathbf{x})$ and $\sigma_i(\mathbf{x})$.

Assuming the random variables $\log S_i(\mathbf{x})$ and $\log S_0(\mathbf{x})$ are independent we state that

$$\mathbb{E}\left\{ \left(\log \frac{S_i(\mathbf{x})}{S_0(\mathbf{x})} \right)^2 \right\} = \mathbb{E}\left\{ (\log S_i(\mathbf{x}))^2 \right\} + \mathbb{E}\left\{ (\log S_0(\mathbf{x}))^2 \right\} - 2\,\mathbb{E}\{\log S_i(\mathbf{x})\}\,\mathbb{E}\{\log S_0(\mathbf{x})\}.$$

Given the asymptotic expansion of the expectation $\mathbb{E}\{(\log S_i(\mathbf{x}))^2\}$ we revise the the $\mathrm{RTOP}^{(2)}$ formulation to handle the non-stationary log-Rician statistics

New solution

$$\mathrm{RTOP}^{(2)}(\mathbf{x}) = \frac{15}{8} C_\tau b^{3/2} \frac{\left\langle (\log E(\mathbf{q}_i))^2 \right\rangle - \mathcal{B}(\mathbf{x})}{\langle -\log E(\mathbf{q}_i) \rangle^{7/2}} - \frac{7}{8} C_\tau b^{3/2} \langle -\log E(\mathbf{q}_i) \rangle^{-3/2}$$

with $\mathcal{B}(\mathbf{x})$ being the bias correction factor

$$\mathcal{B}(\mathbf{x}) = \left\langle \frac{\sigma_i^2(\mathbf{x})}{A_i^2(\mathbf{x})} \right\rangle + \frac{\sigma_0^2(\mathbf{x})}{A_0^2(\mathbf{x})} - \frac{4}{3} \left\langle \frac{\sigma_i^4(\mathbf{x})}{A_i^4(\mathbf{x})} \right\rangle - \frac{4}{3} \frac{\sigma_0^4(\mathbf{x})}{A_0^4(\mathbf{x})}.$$

Experimental results

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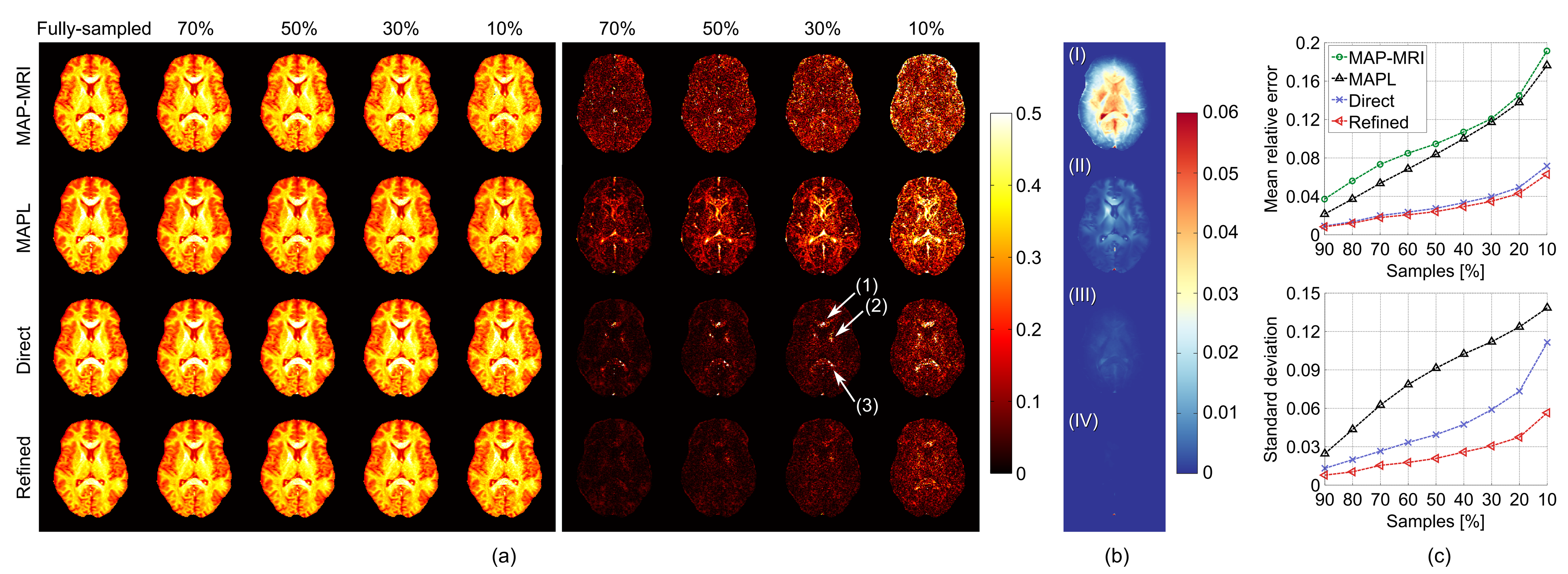

Figure 2: (a) The RTOP measure obtained using the p% samples (left) and the absolute error of the measures with reference to the fully-sampled data. (1) The genu of the corpus callosum (CC), (2) the anterior thalamic radiation and (3) the splenium of the CC. (b) Absolute components of the bias $\mathcal{B}(\mathbf{x})$ for $N_g = 27$. (c) The mean relative error and the standard deviation of the RTOP measure.

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($b = 1000, 3000, 5000, 10000$ s/mm^2)

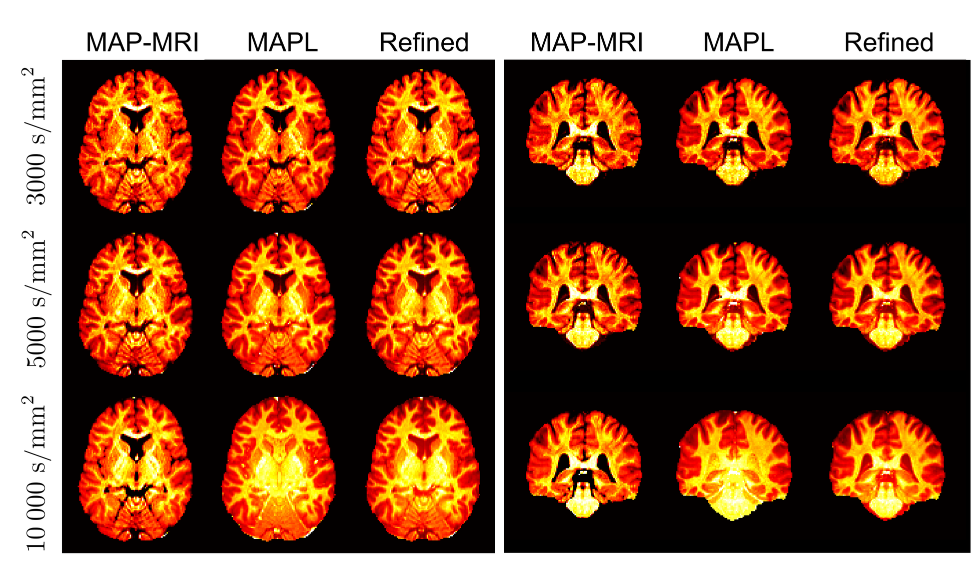

Figure 3: The RTOP measure for maximal b-values.

Table 1: The correlation coefficient between the RTOP measures estimated under different maximal b-values (top) and under different techniques for same maximal b-value (bottom).

	3k/5k s/mm^2	3k/10k s/mm^2	5k/10k s/mm^2
MAP-MRI	0.900	**0.859**	0.942
MAPL	0.876	0.760	0.853
Direct	0.597	0.548	0.639
Refined	**0.929**	0.850	**0.945**

	3k s/mm^2	5k s/mm^2	10k s/mm^2
Refined/MAP-MRI	**0.902**	**0.926**	0.889
Refined/MAPL	0.897	0.904	**0.941**
MAP-MRI/MAPL	0.776	0.842	0.809